6-1 Properties of Exponents

Main Ideas

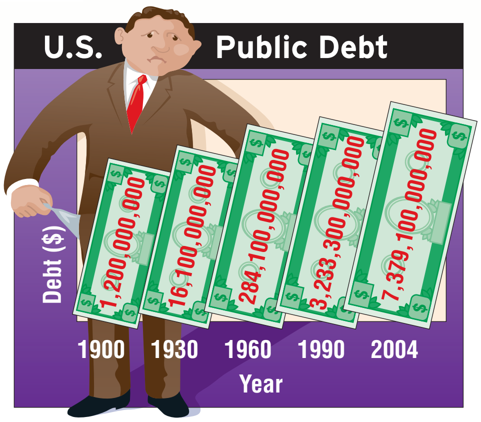
- Use properties of exponents to multiply and divide monomials.
- Use expressions written in scientific notation.

New Vocabulary

simplify
standard notation
scientific notation
dimensional analysis

GET READY for the Lesson

Economists often deal with very large numbers. For example, the table shows the U.S. public debt for several years. Such numbers, written in standard notation, are difficult to work with because they contain so many digits. Scientific notation uses powers of ten to make very large or very small numbers more manageable.



Source: Bureau of the Public Debt

Multiply and Divide Monomials To **simplify** an expression containing powers means to rewrite the expression without parentheses or negative exponents. Negative exponents are a way of expressing the multiplicative inverse of a number. For example, $\frac{1}{x^2}$ can be written as x^{-2}. Note that an expression such as x^{-2} is not a monomial. *Why?*

KEY CONCEPT — *Negative Exponents*

Words For any real number $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.

Examples $2^{-3} = \frac{1}{2^3}$ and $\frac{1}{b^{-8}} = b^8$

Study Tip

Look Back

You can review **monomials** in Lesson 1-1.

EXAMPLE 1 Simplify Expressions with Multiplication

Simplify each expression. Assume that no variable equals 0.

a. $(3x^3y^2)(-4x^2y^4)$

$(3x^3y^2)(-4x^2y^4)$

$= (3 \cdot x \cdot x \cdot x \cdot y \cdot y) \cdot (-4 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y)$ — Definition of exponents

$= 3(-4) \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$ — Commutative Property

$= -12x^5y^6$ — Definition of exponents

b. $(a^{-3})(a^2b^4)(c^{-1})$

$$(a^{-3})(a^2b^4)(c^{-1}) = \left(\frac{1}{a^3}\right)(a^2b^4)\left(\frac{1}{c}\right)$$ Definition of negative exponents

$$= \left(\frac{1}{a \cdot a \cdot a}\right)(a \cdot a \cdot b \cdot b \cdot b \cdot b)\left(\frac{1}{c}\right)$$ Definition of exponents

$$= \left(\frac{1}{\cancel{a} \cdot \cancel{a} \cdot a}\right)(\cancel{a} \cdot \cancel{a} \cdot b \cdot b \cdot b \cdot b)\left(\frac{1}{c}\right)$$ Cancel out common factors.

$$= \frac{b^4}{ac}$$ Definition of exponents and fractions

CHECK Your Progress

1A. $(-5x^4y^3)(-3xy^5)$

1B. $(2x^{-3}y^3)(-7x^5y^{-6})$

Example 1 suggests the following property of exponents.

KEY CONCEPT *Product of Powers*

Words	For any real number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.
Examples	$4^2 \cdot 4^9 = 4^{11}$ and $b^3 \cdot b^5 = b^8$

To multiply powers of the same variable, add the exponents. Knowing this, it seems reasonable to expect that when dividing powers, you would subtract exponents. Consider $\frac{x^9}{x^5}$.

$$\frac{x^9}{x^5} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}}$$ Remember that $x \neq 0$.

$$= x \cdot x \cdot x \cdot x$$ Simplify.

$$= x^4$$ Definition of exponents

It appears that our conjecture is true. To divide powers of the same base, you subtract exponents.

KEY CONCEPT *Quotient of Powers*

Words	For any real number $a \neq 0$, and any integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.
Examples	$\frac{5^3}{5} = 5^{3-1}$ or 5^2 and $\frac{x^7}{x^3} = x^{7-3}$ or x^4

Study Tip

Check

You can check your answer using the definition of exponents.

$$\frac{p^3}{p^8} = \frac{\overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}}}{\underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot p \cdot p \cdot p \cdot p \cdot p}$$

or $\frac{1}{p^5}$

EXAMPLE 2 Simplify Expressions with Division

Simplify $\frac{p^3}{p^8}$. Assume that $p \neq 0$.

$$\frac{p^3}{p^8} = p^{3-8}$$ Subtract exponents.

$$= p^{-5} \text{ or } \frac{1}{p^5}$$ Remember that a simplified expression cannot contain negative exponents.

CHECK Your Progress

Simplify each expression. Assume that no variable equals 0.

2A. $\frac{y^{12}}{y^4}$

2B. $\frac{15c^5d^3}{-3c^2d^7}$

You can use the Quotient of Powers property and the definition of exponents to simplify $\frac{y^4}{y^4}$, if $y \neq 0$.

Method 1

$\frac{y^4}{y^4} = y^{4-4}$ Quotient of Powers

$= y^0$ Subtract.

Method 2

$\frac{y^4}{y^4} = \frac{\overset{1}{y} \cdot \overset{1}{y} \cdot \overset{1}{y} \cdot \overset{1}{y}}{\underset{1}{y} \cdot \underset{1}{y} \cdot \underset{1}{y} \cdot \underset{1}{y}}$ Definition of exponents

$= 1$ Divide.

In order to make the results of these two methods consistent, we define $y^0 = 1$, where $y \neq 0$. In other words, any nonzero number raised to the zero power is equal to 1. *Notice that 0^0 is undefined.*

The properties we have presented can be used to verify the properties of powers that are listed below.

KEY CONCEPT *Properties of Powers*

Words		Examples
	Suppose a and b are real numbers and m and n are integers. Then the following properties hold.	
	Power of a Power: $(a^m)^n = a^{mn}$	$(a^2)^3 = a^6$
	Power of a Product: $(ab)^m = a^m b^m$	$(xy)^2 = x^2y^2$
	Power of a Quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$ and	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0, b \neq 0$	$\left(\frac{x}{y}\right)^{-4} = \frac{y^4}{x^4}$
	Zero Power: $a^0 = 1$, $a \neq 0$	$2^0 = 1$

Study Tip

Simplified Expressions

A monomial expression is in simplified form when:

- there are no powers of powers,
- each base appears exactly once,
- all fractions are in simplest form, and
- there are no negative exponents.

EXAMPLE Simplify Expressions with Powers

3 **Simplify each expression.**

a. $(a^3)^6$

$(a^3)^6 = a^{3(6)}$ Power of a power

$= a^{18}$ Simplify.

b. $\left(\frac{-3x}{y}\right)^4$

$\left(\frac{-3x}{y}\right)^4 = \frac{(-3x)^4}{y^4}$ Power of a quotient

$= \frac{(-3)^4x^4}{y^4}$ Power of a product

$= \frac{81x^4}{y^4}$ $(-3)^4 = 81$

CHECK Your Progress

3A. $(-2p^3s^2)^5$

3B. $\left(\frac{a}{4}\right)^{-3}$

With complicated expressions, you often have a choice of which way to start simplifying.

EXAMPLE Simplify Expressions Using Several Properties

4 Simplify $\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4$.

Method 1

Raise the numerator and denominator to the fourth power before simplifying.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \frac{(-2x^{3n})^4}{(x^{2n}y^3)^4}$$

$$= \frac{(-2)^4(x^{3n})^4}{(x^{2n})^4\,(y^3)^4}$$

$$= \frac{16x^{12n}}{x^{8n}\,y^{12}}$$

$$= \frac{16x^{12n-8n}}{y^{12}} \text{ or } \frac{16x^{4n}}{y^{12}}$$

Method 2

Simplify the fraction before raising to the fourth power.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \left(\frac{-2x^{3n-2n}}{y^3}\right)^4$$

$$= \left(\frac{-2x^{n}}{y^3}\right)^4$$

$$= \frac{16x^{4n}}{y^{12}}$$

CHECK Your Progress

4A. $\left(\frac{3x^2y}{2xy^4}\right)^3$

4B. $\left(\frac{-3x^{-5}y^{-2n}}{5x^{-6}}\right)^4$

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Scientific Notation The form that you usually write numbers in is **standard notation.** A number is in **scientific notation** when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. Real-world problems using numbers in scientific notation often involve units of measure. Performing operations with units is known as **dimensional analysis.**

Real-World EXAMPLE

5 **ASTRONOMY** **After the Sun, the next-closest star to Earth is Alpha Centauri C, which is about 4×10^{16} meters away. How long does it take light from Alpha Centauri C to reach Earth? Use the information at the left.**

Begin with the formula $d = rt$, where d is distance, r is rate, and t is time.

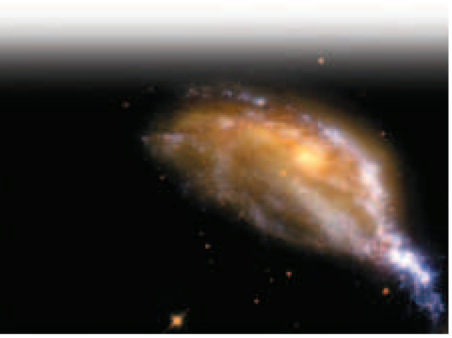
$t = \frac{d}{r}$ — Solve the formula for time.

$= \frac{4 \times 10^{16}\text{ m}}{3.00 \times 10^8\text{ m/s}}$ ← Distance from Alpha Centauri C to Earth

$= \frac{4}{3.00} \cdot \frac{10^{16}}{10^8} \cdot \frac{\text{m}}{\text{m/s}}$ — **Estimate:** The result should be slightly greater than $\frac{10^{16}}{10^8}$ or 10^8.

$\approx 1.33 \times 10^8\text{ s}$ — $\frac{4}{3.00} \approx 1.33$, $\frac{10^{16}}{10^8} = 10^{16-8}$ or 10^8, $\frac{\text{m}}{\text{m/s}} = \text{m} \cdot \frac{\text{s}}{\text{m}} = \text{s}$

It takes about 1.33×10^8 seconds or 4.2 years for light from Alpha Centauri C to reach Earth.



Real-World Link

Light travels at a speed of about 3.00×10^8 m/s. The distance that light travels in a year is called a *light-year.*

Source: www.britannica.com

CHECK Your Progress

5. The density D of an object in grams per milliliter is found by dividing the mass m of the substance by the volume V of the object. A sample of platinum has a mass of 8.4×10^{-2} kilogram and a volume of 4×10^{-6} cubic meter. Use this information to calculate the density of platinum.

Check Your Understanding

Simplify. Assume that no variable equals 0.

Examples 1, 2 (pp. 312–313)

1. $(-3x^2y^3)(5x^5y^6)$ **2.** $\frac{30y^4}{-5y^2}$ **3.** $\frac{-2a^3b^6}{18a^2b^2}$

Example 3 (p. 314)

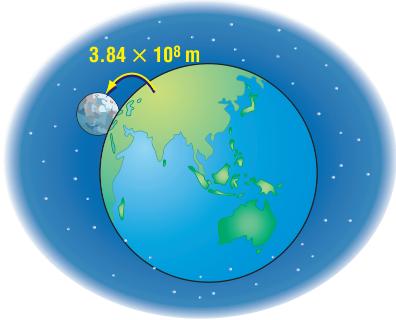
4. $(2b)^4$ **5.** $\left(\frac{1}{w^4z^2}\right)^3$ **6.** $\left(\frac{cd}{3}\right)^{-2}$

Example 4 (p. 315)

7. $(n^3)^3(n^{-3})^3$ **8.** $\frac{81p^6q^5}{(3p^2q)^2}$ **9.** $\left(\frac{-6x^6}{3x^3}\right)^{-2}$

Example 5 (p. 315)

10. ASTRONOMY Refer to Example 5 on page 315. The average distance from Earth to the Moon is about 3.84×10^8 meters. How long would it take a radio signal traveling at the speed of light to cover that distance?



Exercises

HOMEWORK HELP	
For Exercises	See Examples
11–14	1
15–18	2
16–19	3
23–26	4
27, 28	5

Simplify. Assume that no variable equals 0.

11. $\left(\frac{1}{3}a^8b^2\right)(2a^2b^2)$ **12.** $(5cd^2)(-c^4d)$ **13.** $(7x^3y^{-5})(4xy^3)$

14. $(-3b^3c)(7b^2c^2)$ **15.** $\frac{a^2n^6}{an^5}$ **16.** $\frac{-y^5z^7}{y^2z^5}$

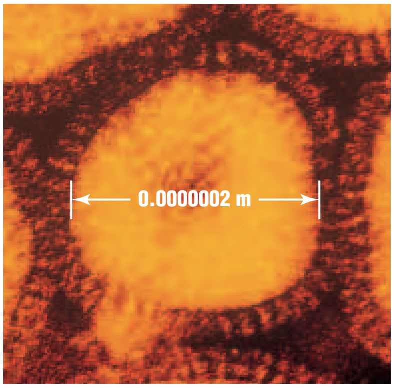
17. $\frac{-5x^3y^3z^4}{20x^3y^7z^4}$ **18.** $\frac{3a^5b^3c^3}{9a^3b^7c}$ **19.** $(n^4)^4$

20. $(z^2)^5$ **21.** $(2x)^4$ **22.** $(-2c)^3$

23. $(a^3b^3)(ab)^{-2}$ **24.** $(-2r^2s)^3(3rs^2)$

25. $\frac{2c^3d(3c^2d^5)}{30c^4d^2}$ **26.** $\frac{-12m^4n^8\,(m^3n^2)}{36m^3n}$

27. BIOLOGY Use the diagram at the right to write the diameter of a typical flu virus in scientific notation. Then estimate the area of a typical flu virus. (*Hint:* Treat the virus as a circle.)



28. POPULATION The population of Earth is about 6.445×10^9. The land surface area of Earth is 1.483×10^8 km^2. What is the population density for the land surface area of Earth?

Simplify. Assume that no variable equals 0.

29. $2x^2(6y^3)(2x^2y)$ **30.** $3a(5a^2b)(6ab^3)$ **31.** $\frac{30a^{-2}b^{-6}}{60a^{-6}b^{-8}}$

32. $\frac{12x^{-3}y^{-2}z^{-8}}{30x^{-6}y^{-4}z^{-1}}$ **33.** $\left(\frac{x}{y^{-1}}\right)^{-2}$ **34.** $\left(\frac{v}{w^{-2}}\right)^{-3}$

35. $\left(\frac{8a^3b^2}{16a^2b^3}\right)^4$ **36.** $\left(\frac{6x^2y^4}{3x^4y^3}\right)^3$ **37.** $\left(\frac{4x^{-3}y^2}{xy^{-5}}\right)^{-2}$

38. If $2^{r+5} = 2^{2r-1}$, what is the value of r?

39. What value of r makes $y^{28} = y^{3r} \cdot y^7$ true?

EXTRA PRACTICE
See pages 902, 931.

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40. INCOME In 2003, the population of Texas was about 2.21×10^7. The personal income for the state that year was about 6.43×10^{11} dollars. What was the average personal income?

41. RESEARCH Use the Internet or other source to find the masses of Earth and the Sun. About how many times as large as Earth is the Sun?

H.O.T. Problems

42. OPEN ENDED Write an example that illustrates a property of powers. Then use multiplication or division to explain why it is true.

43. FIND THE ERROR Alejandra and Kyle both simplified $\frac{2a^2b}{(-2ab^3)^{-2}}$. Who is correct? Explain your reasoning.

Alejandra

$$\frac{2a^2b}{(-2ab^3)^{-2}} = (2a^2b)(-2ab^3)^2$$
$$= (2a^2b)(-2)^2a^2(b^3)^2$$
$$= (2a^2b)2^2a^2b^6$$
$$= 8a^4b^7$$

Kyle

$$\frac{2a^2b}{(-2ab^3)^{-2}} = \frac{2a^2b}{(-2)^2a(b^3)^{-2}}$$
$$= \frac{2a^2b}{4ab^{-6}}$$
$$= \frac{2a^2bb^6}{4a}$$
$$= \frac{ab^7}{2}$$

44. REASONING Determine whether $x^y \cdot x^z = x^{yz}$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

45. CHALLENGE Determine which is greater, 100^{10} or 10^{100}. Explain.

46. *Writing in Math* Use the information on page 312 to explain why scientific notation is useful in economics. Include the 2004 national debt of \$7,379,100,000,000 and the U.S. population of 293,700,000, both written in words and in scientific notation, and an explanation of how to find the amount of debt per person with the result written in scientific notation and in standard notation.

STANDARDIZED TEST PRACTICE

47. ACT/SAT Which expression is equal to $\frac{(2x^2)^3}{12x^4}$?

A $\frac{x}{2}$

B $\frac{2x}{3}$

C $\frac{1}{2x^2}$

D $\frac{2x^2}{3}$

48. REVIEW Four students worked the same math problem. Each student's work is shown below.

Student F

$$x^2x^{-5} = \frac{x^2}{x^5} = \frac{1}{x^3}, x \neq 0$$

Student G

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^7, x \neq 0$$

Student H

$$x^2x^{-5} = \frac{x^2}{x^{-5}} = x^{-7}, x \neq 0$$

Student J

$$x^2x^{-5} = \frac{x^2}{x^5} = x^3, x \neq 0$$

Which is a completely correct solution?

F Student F

G Student G

H Student H

J Student J

Spiral Review

Solve each inequality algebraically. (Lesson 5-8)

49. $x^2 - 8x + 12 < 0$ **50.** $x^2 + 2x - 86 \geq -23$ **51.** $15x^2 + 4x + 12 \leq 0$

Graph each function. (Lesson 5-7)

52. $y = -2(x - 2)^2 + 3$ **53.** $y = \frac{1}{3}(x + 5)^2 - 1$ **54.** $y = \frac{1}{2}x^2 + x + \frac{3}{2}$

Evaluate each determinant. (Lesson 4-3)

55. $\begin{vmatrix} 3 & 0 \\ 2 & -2 \end{vmatrix}$ **56.** $\begin{vmatrix} 1 & 0 & -3 \\ 2 & -1 & 4 \\ -3 & 0 & 2 \end{vmatrix}$

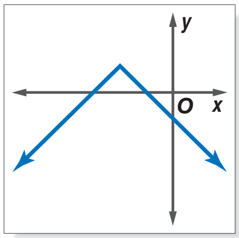
Solve each system of equations. (Lesson 3-5)

57. $x + y = 5$
$x + y + z = 4$
$2x - y + 2z = -1$

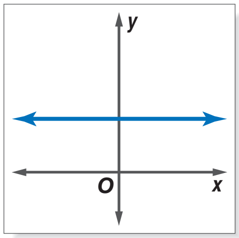
58. $a + b + c = 6$
$2a - b + 3c = 16$
$a + 3b - 2c = -6$

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. (Lesson 2-6)

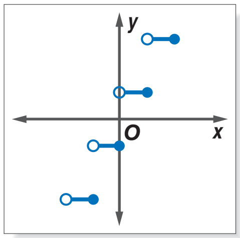
59.



60.



61.

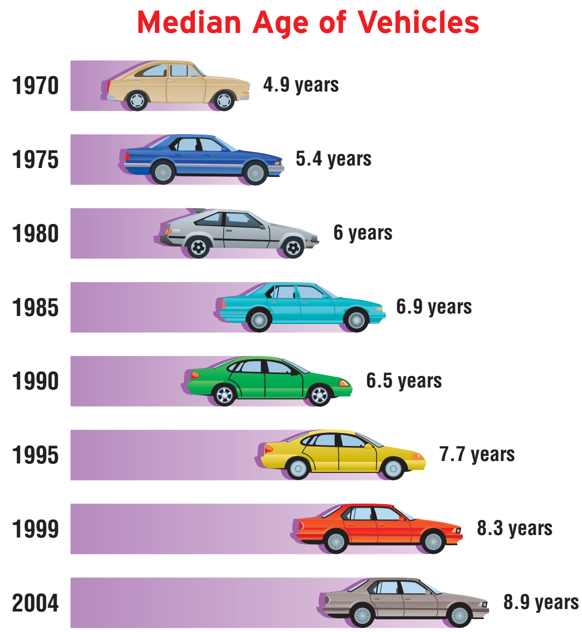


TRANSPORTATION For Exercises 62–64, refer to the graph at the right. (Lesson 2-5)

62. Make a scatter plot of the data, where the horizontal axis is the number of years since 1975.

63. Write a prediction equation.

64. Predict the median age of vehicles on the road in 2015.



Source: Transportation Department

Solve each equation. (Lesson 1-3)

65. $2x + 11 = 25$

66. $-12 - 5x = 3$

GET READY for the Next Lesson

PREREQUISITE SKILL Use the Distributive Property to find each product. (Lesson 1-2)

67. $2(x + y)$ **68.** $3(x - z)$ **69.** $4(x + 2)$

70. $-2(3x - 5)$ **71.** $-5(x - 2y)$ **72.** $-3(-y + 5)$